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Vector Sparse Representation of Color Image Using Quaternion Matrix Analysis

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Abstract— Traditional sparse image models treat color image pixel as a scalar, which represents color channels separately or concatenate color channels as a monochrome image. In this paper, we propose a vector sparse representation model for color images using quaternion matrix analysis. As a new tool for color image representation, its potential applications in several image-processing tasks are presented, including color image reconstruction, denoising, inpainting, and super-resolution. The proposed model represents the color image as a quaternion matrix, where a quaternion-based dictionary learning algorithm is presented using the K-quaternion singular value decomposition (QSVD) method. It conducts the sparse basis selection in quaternion space, which uniformly transforms the channel images to an orthogonal color space.

Keywords—Vector sparse representation, quaternion matrix analysis, color image, dictionary learning, K-QSVD, image restoration

I. INTRODUCTION

The theory of sparse representation has been proven as an effective model for image representation. Using an overcomplete dictionary that contains a certain number of prototype atoms as its elements, an image signal can be represented as a sparse linear combination of these atoms. The performance of sparse coding relies on the quality of dictionary, which could be chosen as a pre-defined set of bases, such as overcomplete wavelets, curvelets, contourlets etc. As for color images, the sparse model treat RGB channels as three independent "gray-scale" images and process them in a monochrome way. These works completely ignore the inter-relationship among the multiple channels, and produces hue distortions in the reconstruction results. To avoid color distortions, some works has been proposed to concatenate RGB channels, where a dictionary is trained to jointly represent the channels.

II. QUATERNION-BASED DICTIONARY TRAINING APPROACH

2.1 Quaternion Orthogonal Matching Pursuit Algorithm

1. Initialization: Residual $\dot{\varepsilon}(0) = \dot{y}$, atom set $\dot{S}^{(0)}=\Phi$.
2. For j=I~L:

1) For every $d_m \in \dot{D} \setminus \dot{S}^{(j-1)}$,compute correlation:
   $C_m^{(j)} = < d_m, \dot{\varepsilon}^{(j-1)} >= d_m^H \dot{\varepsilon}^{(j-1)}$
2) Atom selection: $m^{(j)} = \mathrm{argmax}_m \|C_m^{(j)}\|$, $\dot{d}^{(j)} = [\dot{D} \setminus \dot{S}^{(j-1)}]_{m^{(j)}}$, $\dot{S}^{(j)} = \dot{S}^{(j-1)} \cup \dot{d}^{(j)}$
3) Compute coefficient: $\dot{a}^{(j)} = ((\dot{S}^{(j)})^H \dot{S}^{(j)})^{-1} (\dot{S}^{(j)})^H \dot{y} = (\dot{S}^{(j)}) \dagger \dot{y}$
4) Residual update: $\dot{\varepsilon}^{(j)} = \dot{y} - \dot{S}^{(j)} \dot{a}^{(j)}$

2.2 Steps of algorithm

1) We initialize the residual $\dot{\varepsilon}(0) = \dot{y}$ as the input patch $\dot{y}$ itself, and the atom set $\dot{S}$ as an empty set.

2) At the j-th iteration, QOMP selects the atom that produces the largest projection onto current residual. First, we compute the correlation between current residual and each atom d˙m from the atom pool D˙ \ S˙(j−1) , i.e., C(j) m = < d˙ m, ε˙(j−1) >. Then we add the atom which achieves the highest correlation value into atom set S˙(j) .
3) We compute coefficients by˙(j) = ((S˙(j))H S˙(j))−1(S˙(j))H y˙ = (S˙(j))†y˙, where the superscript † denotes the quaternionic pseudoinverse operation. ((S˙(j))H S˙(j))−1 is calculated as follows: we first compute the QSVD of (S˙(j))H S˙(j) , then replace all nonzero singular values by their reciprocals.
4) We refine the residual signal as ε˙(j) = ˙y − S˙(j) a˙(j).

III. QUATERNION-BASED DICTIONARY LEARNING APPROACH

3.1 K-QSVD Algorithm

1. **Initialization:** Construct the training color data Y= {˙y,1 ≤ i ≤ N}, and initialize the dictinory matrix D˙ ={ d_i˙ ,1 ≤ i ≤ K} as random samples from Y, where each atom d_i˙Є H^n and block patch y_i € H^n .
2. **Repeate J times:**
   1) Sparse Coding Stage: Use QOMP to compare the coefficient matrix A˙={α_i ,1 ,1 ≤ i ≤ N},where each coefficient column α_i € H^K .
   2) Code Book Update Stage: Update each dictionary atom d_k˙ in $D^{\cdot J}$ through (i)-(iii) s teps.
      (i) Find the set of patches that use atom d_k ,the index w_k={i|1≤ i ≤ N, A(k,i) ≠ 0}, where A(k,i) indicates the entry at k-th row and i-th column of the coefficient matrix A.
      (ii) Compute the error $E^k = Y - \Sigma_{j\neq k} d_j{}^{\cdot}A^j$ and select the columns corresponding to w_k to from $E_k^R = E_k(i,j)|_{j=wk}$ for QSVD: E_k^R = U^V^H.
      (iii) Updates d_k as the first column vector of U, and set it s corresponding nonzero coefficient $a_R^k = w_k^T a^k$ to be the muiltification of the first column of v^H and ^(1,1).

3.2 Steps of algorithm

1) We initialize the dictionary matrix D as random samples from training data Y.
2) Sparse Coding Stage: Compute the co-efficient matrix using QOMP.
   Codebook Update Stage: For each atom ˙**d**_k_ and the corresponding coefficients ˙**A**_k_ - the _k_-th row of ˙**A**, we update both of them by decomposing the remaining representation error $E^k = Y - \Sigma_{j\neq k} d_j{}^{\cdot}A^j$ using QSVD.

IV. RELATED RESEARCH WORK

4.1 Robust face recognition via sparse representation

Here the problem of automatically recognizing human faces is considered from frontal views with varying expression and illumination, as well as occlusion and disguise. Sparse signal representation offers the key to addressing this problem based on a sparse representation computed by C^1-minimization, hence a general classification algorithm for (image-based) object recognition was proposed. Issues in face recognition: feature extraction and robustness to occlusion.

4.2 K-SVD: An algorithm for designing overcomplete dictionaries for sparse representation

Applications that use sparse representation are many and include compression, regularization in inverse problems, feature extraction, and more. Recent activity in this field has concentrated mainly on the study of pursuit algorithms that decompose signals with respect to a given dictionary. Given a set of training signals, we seek the dictionary that leads to the best representation for each member in this set, under strict sparsity constraints. K-SVD is an iterative method that alternates between sparse coding of the examples based on the current dictionary and a process of updating the dictionary atoms to better fit the data.

4.3 Image denoising via sparse and redundant representations over learned dictionaries

In image denoising problem, where zero-mean white and homogeneous Gaussian additive noise is to be removed from a given image. Using the K-SVD algorithm, we obtain a dictionary that describes the image content effectively. Two training options are considered: using the corrupted image itself, or training on a corpus of high-quality image database.

V. EXISTING SYSTEM

In recent years, the learning based strategies for designing are proposed to represent input signals more sparsely. Structures low rank representations for signal classification have attracted much attention of researchers. Compact and discriminative dictionary is learned to include structure information based on the results of a linear predict classifier. Meanwhile, the new concepts of block sparsity and group sparsity are defined to get more structural coefficients for different classes. There is a lack of general model and technique for color image analysis and processing.

VI. PROPOSED SYSTEM

In this work, we propose a novel sparse model for color image using quaternion matrix analysis. It formulates a color pixel as a vector unit instead of a scalar quantity and consequently overcomes the lack of accuracy describing inter-relationship among color channels. The experiments of reconstruction, de-noising, in-painting and super-resolution on natural color images prove its advantages in effectively accounting for both luminance and chrominance geometry in images. Currently, the usage of the real part of quaternion seems insufficient: for three-channel color space, the real part is simply set to be zero. We belive that the physically meaningful real part will further help us capture color information. In the future, we will further explore the potential extension of quaternion sparse model to four channel color space, e.g. CMYK, in which the real part may correspond to the black channel. Additionaly, from the view of algorithm our K-QSVD algorithm does not guarantee global convergence.

VII. CONCLUSION AND FUTURE ENHANCEMENTS

In this project, we propose a novel sparse model for color image using quaternion matrix analysis. It formulates a color pixel as a vector unit instead of a scalar quantity and consequently overcomes the lack of accuracy describing inter-relationship among color channels. The experiments of denoising, inpainting, and super-resolution on natural color images prove its advantages in effectively accounting for both luminance and chrominance geometry in images.

Currently, the usage of the real part of quaternion seems insufficient: for three-channel color space, the real part is simply set to be zero. In the future, we will further explore the potential extension of quaternion sparse model to four channel color space, e.g. CMYK, in which the real part may corresponds to the black channel.

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